The Report Committee for Xiaojie Wang
Certifies that this is the approved version of the following report:

Introduction to Statutory Reserves in Life Insurance Companies

APPROVED BY
SUPERVISING COMMITTEE:

Supervisor:

Mark M Maxwell

Supervisor:

Peter Mueller

Su Long

Introduction to Statutory Reserves in Life Insurance Companies

by

Xiaojie Wang, B.S.

Report

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Arts

The University of Texas at Austin

May 2011

Acknowledgements

I would like to thank Mr. Su Long (Actuarial Analyst and Portfolio Analyst) and Mr. James B Hobelman (FSA, Head of Actuary) from New Era Life Insurance Companies for their education on statutory reserve valuations. They helped me on understanding statutory reserve valuation laws and methods and showed me how the calculation system worked.

I also would like to thank Dr. Mark M Maxwell and Dr. Peter Mueller for their precious time and kind efforts to read my report, give me helpful advices and to be my report committee members.

Last but not the least; I would like to thank my family for their tremendous love and support.

Abstract

Introduction to Statutory Reserves in Life Insurance Companies

Xiaojie Wang, M.A.

The University of Texas at Austin, 2011

Supervisor: Mark M Maxwell, Peter Mueller

Statutory reserves in life insurance companies are required by regulation laws. Regulators monitor insurers' statutory reserves to protect policy holders' future benefits and ensure the insurers are financially healthy. The purpose of this report is to give a brief introduction to statutory reserves in life insurance companies. In this report, assumptions and valuation methods for statutory reserve valuations are explained and discussed. The comparisons between statutory reserves and GAAP reserves are also discussed.

Table of Contents

List of Tables

List of Figures

Chapter One: Significance of Insurance

We are all exposed to a variety of risks (potential loss) in our lifetime. Some of the risks would eventually cause losses. When unexpected losses occur, people need money to cover their losses. The amount of money that they need would range anywhere from hundreds of dollars to even a few million dollars. For example, in the United States a pregnant woman without insurance coverage will need approximately $6,000-$8,000 on average for delivery alone [1] and more than 10,000 US dollars for both prenatal visits and maybe at least 3-month living expense which depends on the living standard of her residential area. With healthcare insurance, up to 50% of these pregnancy expenses could be covered.

Without the coverage by insurance, individual or companies will have to set aside a large amount of money to prepare for the occurrence of losses which no one would know exactly the timing or severity. Insurance allows people and employers to transfer their risks to insurers by paying a relatively smaller amount of premiums compared to their uncertain future losses, which brings them peace of mind and enhances their financial strength. That is why insurance is beneficial.

A sentence from Statement of Statutory Accounting Principles describes the purpose of insurance very well which is quoted here for your reference:

"*The primary purpose of insurance, including managed care coverage, is to provide economic protection from identified risks occurring or discovered within a specified period*."

[1] pregnancy statistical data cited from American Pregnancy Association website.
http://www.americanpregnancy.org/planningandpreparing/affordablehealthcare.html

Chapter Two: Importance of Reserves for Insurance Companies

Reserve is the amount of money that insurance companies have to hold in order to cover payments of future claims, unearned premiums in case that some policy holders would cancel their contracts before their coverage periods end (unearned premium is explained in Section 5.1), and etc. This reserve requirement is not only for risk management but also heavily regulated by Department of Insurance in every state of American.

For example, a life insurance company had one hundred policies issued at the beginning of 2011. Without considering meeting the reserve requirements, the company could count all the collected premiums as its revenue at the end of 2011 and then take profit based on this revenue. Typically nothing goes wrong at the beginning since the expected frequency of claims is usually small after policies just started. However, as time goes by, the expected frequency of claims usually increases and eventually in one year the claims costs are going to exceed the total annual premiums collected. From then on, the life insurance company would need reserve an amount of cash to offset the extra claims costs and starts to have negative profit. This volatile cycle (huge gains at the beginning and huge losses at the end) is apparently risky and unhealthy for insurance companies. In another word, if all insurance companies operate without reserves, one would ask where the risk management comes to play. In reality, insurance companies will set aside part of their collected annual premiums as reserve to pay future claims. If an insurance company doesn't prepare for enough reserve, it will face the insolvency crisis when claim payments come due at a certain point of time in the near future. Inaccurate reserves, either underestimated or overestimated, will impact the financial health of insurance companies. Overestimated reserve will reduce the amount of profit that an insurance company should be able to gain and invest for assets growth. Therefore, insurance companies hire qualified

actuaries to calculate the net present value of expected future premiums and claims using actuarial models.

Chapter Three: Statutory Reserves vs. GAAP Reserves

3.1 Statutory Reserves

It has been about two centuries since the first life insurance company started in the United States. Statutory reserves were required by state insurance regulators as generally accepted reserve methods for life insurance companies. Regulators monitor insurers' statutory reserves to protect policy holders' future benefits and ensure the insurers are financially healthy. Statutory reserves employ conservative valuation methods, which use higher mortality rates and lower interest rates, compared to GAAP reserves. Also, future withdrawal rate (or lapse rate) is taken into account in GAAP reserves calculations but not in Statutory reserves calculations although the withdrawals will usually reduce the amount of future claims to be paid and thus the reserves needed today. The valuation standards for minimum reserves that an insurance company must hold in the United States are Commissioners Reserves Valuation method (CRVM) for life insurance plans and Commissioners Annuity Reserve Valuation Method (CARVM) for annuities (CRVM will be described in Section 7.4 and CARVM will not be discussed here).

3.2 GAAP Reserves

Prior to early 1970s, the statutory accounting practices had been widely used for all businesses of insurance industry. However the problems of using statutory reserve method are that life insurance companies' first-year profits for new policies are usually negative due to large percentage of commissions and other expenses which usually occur during the first policy year. As a result, the more policies that a company sells, the more losses the company will have during the first policy year. This issue was not addressed well until more and more insurance companies became publicly traded and the owners and investors

found out the information from statutory reserves could be misleading. Investors would need more ways to evaluate the earning performance of life insurance companies from business point of views not just from the regulatory point of views. Generally Accepted Accounting Principles (GAAP) in the United States defined the accounting rules and standards that companies present on financial statement. GAAP focuses on the earnings and enterprise performance, which provides the reasonable understandings of business activities. Then publicly owned insurance companies started to apply general accepted accounting principle (GAAP) for their financial reports. In 1972, Audits of Stock Life insurance Companies (Audit Guide) introduced the GAAP as the accounting standards for life insurance companies. After that, series of accounting pronouncements were released to guide life insurance industry. There were three important Statements of Financial Accounting Standards applied to life insurers: SFAS 60 in 1982, SFAS 97 in 1987, and SFAS 113 in 1992. Compared to statutory reserves valuations for which actuaries need to adhere to many limits in selecting mortality rates and interest rates by regulation rules, actuaries could have greater flexibility in GAAP valuations. In order to avoid the negative earnings during the first year of new policies sales, deferred Acquisition Cost method is adopted by GAAP for policy expenses.

Chapter Four: Valuation Requirement: Standard Valuation Law

The earlier version of Standard Valuation Law (SVL) defined the minimum reserves using net level premium method by specifying the mortality tables and interest rates. Later on, insurance companies found that the first year expense would exceed the collected annual premium and insurance companies would be unable to take gains or set aside some money to establish the reserve. In the 1940's, SVL was amended by adopting the Commissioners Reserve Valuation Method (CRVM) as the reserve method to address this issue. Details will be discussed in section 7.4. In 1980, SVL required insurance companies to use 1980 Commissioner Standard Ordinary Mortality table as minimum reserves standard.

Chapter Five: Types of Reserves

Reserves can generally be classified into three categories.

5.1 Premium Reserves

If a policy holder pays annual premium at the beginning of the policy year, this policy holder's insurance company would not earn the entire annual premium until the end of that year when the entire policy year's insurance coverage has been applied. Suppose the actuary valuates the reserve in the middle of the policy year, half of the annual premium has been earned and the other half will be treated as unearned premium. At the end of the policy year, the insurer will then earn the entire annual premium. Therefore, insurers are required to set up premium reserve to assure the refund of unearned portion of premiums if policy holders want to cancel their policies before the end of coverage period.

Unearned premium liability is calculated on the net basis in most companies. For example, according to "Valuation of Life Insurance Liabilities", if the premium mode is monthly, then unearned premium liability is

$$\frac{k}{m'} \cdot {}_{m}P_{x+t:n}^{(m')}$$

x: *issue age;*

t: *policy duration;*

k: *the number of months to the next premium due date;*

m': *the number of months between two closest due date*;

${}_{m}P_{x+t:n}^{(m')}$: *The annual net premium at time t.*

5.2 Contract Reserves

When policy holders buy insurance products, the benefits have already been defined in insurance contracts and will be paid when certain conditions are met, for example, death, disability, etc. Reserve should be established to pay the future benefits when claims are made. Contract reserves are generally calculated by subtracting the present value of future premiums from the present value of future benefits (See Chapter Seven).

5.3 Claim Reserves

Claim reserves are established to cover those potential payments when claims have incurred during the coverage period but have not yet been paid. IBNR is short for “incurred but not reported” and is the alternative name for claim reserves. A number of methods exist for IBNR calculation. One of those methods is to use past experience data and statistical models to estimate the future claim reserves.

Chapter Six: Statutory Reserves Assumptions

6.1 Mortality Table

Since 1940s, mortality tables have been updated once per twenty years due to the increase of life expectancy. The 1958 mortality tables only represented the male mortality rates and actuaries had to use males' mortality rates for females. What they did was to set three-year age back to reflect the female's lower mortality rate (for example, used the mortality rate of males at age of 46 for females at age of 49), which could not well approximate the male-female difference in mortality rates across the age spectrum. The reduction in mortality rates for insured of all ages resulted in the adoption of 1980 Commissioner Standard Ordinary Mortality table (1980 CSO Mortality Tables) as minimum reserves standard in the 1980 Amendments to SVL. The 1980 Tables not only had both male and female classifications but also smoker and non-smoker statuses. Unisex category with and without smoking statuses were also included in the 1980 Tables.

During the development of 1980 tables, most of the experience data were collected based on age nearest birthday (ANB) [1]. The 1980 CSO tables of age last birthday (ALB) [2] were derived from the 1980 CSO tables of age nearest birthday (ANB) by apply the following formula:

$$q_x^{ALB} = [\, q_x + \, (1 - q_x)q_{x+1}]/(2 - q_x)$$

[1] Age nearest birthday refers to the age at the nearest birthday to the valuation day (could be last birthday or next birthday);
[2] Age last birthday refers to the age of last birthday only.

For life policies issued after January 1st, 2004, the 2001 CSO Tables may be used for reserve valuations. After January 1st, 2009, new issued policies must

apply the 2001 CSO Tables for reserve valuations. The 2001 CSO Tables extended the maximum age to 120 instead of 100 in 1980 CSO tables.

6.2 Interest Rate

Reserves are generally equal to present value of excess future benefits over future premiums. Therefore, generally the higher interest rates are used, the less the reserves would be. Maximum interest rates one could use in the reserve valuations were defined by the 1980 Amendment to SVL. In 1984, the current bond index rates were introduced to calculate the interest rate. It varies by policy durations which are the maximum number of years for life policies remaining in force. The maximum interest rate for whole life policies has been 4% since 2006 (See column ">20 years" in Table 2).

According to SVL, life insurance companies must determine the interest rates for statutory reserves by using the following formula *(according to standard valuation law)*:

$$I = 0.03 + W(R_1 - 0.03) + \frac{W}{2}(R_2 - 0.09)$$

R1 is the less of R and 0.09;

R2 is the greater of R and 0.09;

R is the reference interest rate calculated using averaged Moody's corporate bond yields over a certain period of time;

W is the weighting factor defined as the following table.

Table 1 Weighting Factors for Interest Rate

Guarantee Duration	Weighting Factor
< 10 years	0.50
10 ~20 years	0.45
>20 years	0.35

Table 2

Historical Maximum Valuation Interest Rates and Mortality Tables

Issue year	< 10 years	10~ 20 years	> 20 years	CSO table
1983	7.25	6.75	6.00	CSO 80 (sex-distinct)
1984	7.25	6.75	6.00	CSO 80 (sex-distinct)
1985	7.25	6.75	6.00	CSO 80 (sex-distinct)
1986	7.25	6.75	6.00	CSO 80 (sex-distinct, smoking optional)
1987	6.50	6.00	5.50	CSO 80 (sex-distinct, smoking optional)
1988	6.00	6.00	5.50	CSO 80 (sex-distinct, smoking optional)
1989	6.00	6.00	5.50	CSO 80 (sex-distinct, smoking optional)
1990	6.00	6.00	5.50	CSO 80 (sex-distinct, smoking optional)
1991	6.00	6.00	5.50	CSO 80 (sex-distinct, smoking optional)
1992	6.00	6.00	5.50	CSO 80 (sex-distinct, smoking optional)
1993	6.00	6.00	5.00	CSO 80 (sex-distinct, smoking optional)
1994	5.50	5.25	5.00	CSO 80 (sex-distinct, smoking optional)
1995	5.50	5.25	4.50	CSO 80 (sex-distinct, smoking optional)
1996	5.50	5.25	4.50	CSO 80 (sex-distinct, smoking optional)
1997	5.50	5.25	4.50	CSO 80 (sex-distinct, smoking optional)
1998	5.50	5.25	4.50	CSO 80 (sex-distinct, smoking optional)
1999	5.00	4.75	4.50	CSO 80 (sex-distinct, smoking optional)
2000	5.00	4.75	4.50	CSO 80 (sex-distinct, smoking optional)
2001	5.00	4.75	4.50	CSO 80 (sex-distinct, smoking optional)
2002	5.00	4.75	4.50	CSO 80 (sex-distinct, smoking optional)
2003	5.00	4.75	4.50	CSO 80 (sex-distinct, smoking optional)
2004	5.00	4.75	4.50	CSO 80 (sex-distinct, smoking optional)
2005	5.00	4.75	4.50	CSO 80 (sex-distinct, smoking optional)
2006	4.50	4.25	4.00	CSO 80 (sex-distinct, smoking optional)
2007	4.50	4.25	4.00	CSO 80 (sex-distinct, smoking optional)
2008	4.50	4.25	4.00	CSO 80 (sex-distinct, smoking optional)
2009	4.50	4.25	4.00	CSO 01 (sex-distinct, smoking optional)
2010	4.50	4.25	4.00	CSO 01 (sex-distinct, smoking optional)
2011	4.50	4.25	4.00	CSO 01 (sex-distinct, smoking optional)

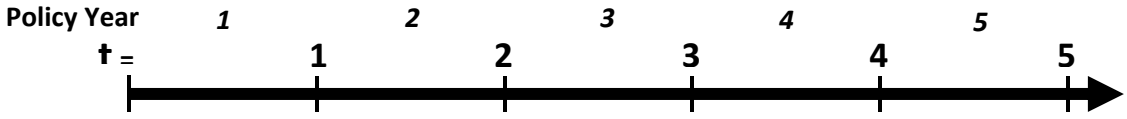
Chapter Seven: Reserve Valuation Methods

7.1 Basic Concepts

Before introducing the following methods for reserve valuation, a few basic concepts are explained here:

7.1.1. Time Line

The policy year is rounded up to the nearest whole number of years between effective policy date and valuation date. And the policy duration equals to policy year less than 1. “t” will be used to represent the policy duration in the following description of reserve methods.



7.1.2. Key Dates

Effective policy date in this report is the date when a policy becomes effective. Maturity date is the date when a policy terminates. Premium payment period is the time period between first premium and last premium. In the following description, “n” will be used for the number of years from effective policy date to maturity date. “m” will be used as the number of years for premium to be paid.

7.2 Net Level Premium Reserve Method

The Net Level Premium means the net premium is always a level percentage of the gross premium. The gross premium is the amount of premium charged by insurance companies (Further explanation, see Chapter 9). Here is the

formula for a net level premium at time t for an m-pay n-year endowment policy.

$$ {}_mPB_{x+t:\overline{n-t}} = \begin{cases} \dfrac{A_{x:\overline{n}}}{\ddot{a}_{x:\overline{m}}} & t = 0 \\ r_{x+t}^{GP} \cdot \dfrac{A_{x:\overline{n}}}{\ddot{a}_{x:\overline{m}}} & 0 < t < m \\ 0 & m \le t < n \end{cases} $$

The gross premium ratio is defined as:

$$ r_{x+t}^{GP} = \frac{GP_{x+t}}{GP_x} $$

GP_x: *The gross premium at time 0;*

GP_{x+t}*: The gross premium at time t.*

If the gross premium is constant during the premium payment period, then the gross premium ratio is equal to 1 ($r_{x+t}^{GP} = 1$ for 0<t<m), which is a net level premium case.

Net level premium reserve for m-pay n-year endowment policy with issue age x at time t is calculated by the following formula:

$$ {}_t^mVB_{x:\overline{n}} = A_{x+t:\overline{n-t}} - {}_mPB_{x:\overline{n}} \cdot \ddot{a}_{x+t:\overline{n-t}} $$

The present value of future benefit at time t by the retrospective formula is:

$$ A_{x+t:\overline{n-t}} = A_{x+t+1:\overline{n-t-1}} \cdot v \cdot p_{x+t} + DB_{x+t} \cdot v \cdot q_{x+t} $$

The present value of annuity due at time t by the retrospective formula is:

$$ \ddot{a}_{x+t:\overline{m-t}} = \ddot{a}_{x+t+1:\overline{m-t-1}} \cdot v \cdot p_x + 1 $$

The net level premium at beginning of the first policy year is:

$$ {}_mPB_{x:\overline{n}} = \frac{A_{x:\overline{n}}}{\ddot{a}_{x:\overline{m}}} $$

This formula shows that the net level premium is only benefit premium and it does not consider any expenses which make it not practical as the ultimate method for reserve calculation. Therefore the reserves under NLP methods are only benefit reserves.

The following table is a calculation example to show net level premium reserves which was calculated by the retrospective formula. First of all, assumptions were presented:

Assume a 40-year-old male, who bought a 10-pay 20-year endowment policy with $100,000 benefit on 1/1/2000. He would pay premiums at the beginning of the first ten years and have $100,000 death benefit unit the end of the 20th policy year. The 1980 CSO mortality table and 4% interest rate are used for reserve valuation.

Here,

Present value of future premium:

$$PVFP_{x+t} = NP_{x+t} + PVFP_{x+t+1} * v * p_{x+t}$$

NP_{x+t}: *the net premium at time* t.

p_{x+t}: *the probability of survival at the age of* $(x+t)$, $p_{x+t} = 1 - q_{x+t.}$

Net reserve:

$$^{m}_{t}VB_{x:\overline{n}} = A_{x+t:\overline{n-t}} - PVFP_{x+t}$$

Table 3

Net level Premium Reserve Method for a 10-pay 20-year Endowment Policy

Pol Yr	Att. Age	t	q(x)	p(x)	DB(x+1)	A(x)	$\ddot{a}(x)$	Net Prem NP(t)	PVFP	VB NLP Rsv
1	40	0	0.0023	0.9977	100,000	47,279	8.3371	5,671	47,279	-
2	41	1	0.0025	0.9975	100,000	49,053	7.6481	5,671	43,372	5,682
3	42	2	0.0027	0.9974	100,000	50,894	6.9311	5,671	39,306	11,588
4	43	3	0.0029	0.9971	100,000	52,805	6.1848	5,671	35,073	17,732
5	44	4	0.0031	0.9969	100,000	54,788	5.4076	5,671	30,666	24,122
6	45	5	0.0033	0.9967	100,000	56,847	4.5980	5,671	26,075	30,772
7	46	6	0.0036	0.9964	100,000	58,985	3.7544	5,671	21,291	37,694
8	47	7	0.0039	0.9961	100,000	61,205	2.8749	5,671	16,303	44,901
9	48	8	0.0042	0.9958	100,000	63,511	1.9575	5,671	11,101	52,410
10	49	9	0.0045	0.9955	100,000	65,909	1.0000	5,671	5,671	60,238
11	50	10	0.0049	0.9951	100,000	68,402	-	-	-	68,402
12	51	11	0.0054	0.9947	100,000	70,995	-	-	-	70,995
13	52	12	0.0059	0.9941	100,000	73,695	-	-	-	73,695
14	53	13	0.0064	0.9936	100,000	76,505	-	-	-	76,505
15	54	14	0.0071	0.9929	100,000	79,433	-	-	-	79,433
16	55	15	0.0078	0.9922	100,000	82,486	-	-	-	82,486
17	56	16	0.0086	0.9914	100,000	85,673	-	-	-	85,673
18	57	17	0.0095	0.9905	100,000	89,005	-	-	-	89,005
19	58	18	0.0104	0.9896	100,000	92,494	-	-	-	92,494
20	59	19	0.0115	0.9885	100,000	96,154	-	-	-	96,154
21	60	20	0.0126	0.9874	-	-	-	-	-	-

7.3 Full-Preliminary Term Reserve Method

NLP method does not consider expenses in the reserve calculations and at the end of the first policy year the life insurance companies would usually have a negative net income (See table 4 below).

In order to 1) include expenses in reserve calculation and also 2) address negative net income issue at the end of the first policy year caused by heavy first year expenses for underwriting life insurance policies, modified reserve methods with expense allowance, such as Full-Preliminary Term Method (FPT), were used.

Expense allowance is applied to reduce the reserve in the first policy year to offset the large first year expenses. And here is detailed explanation:

Expenses and reserves are both considered to be liabilities to insurance companies, which will bring down their net income. If insurance companies could do nothing about heavy first year expenses but they can have a reduced reserve requirement due to expense allowance, the total liability (expense + reduced reserve) in the first policy year will be reduced, which would dramatically increase their net income and probably bring their net income back to be positive (see table 4).

The reduced first year reserve amount will then be amortized in the following premium payment period – i.e., to have increased reserves requirement in the following premium payment period to make up the reduced reserve amount in the first policy year.

Table 4 Comparison between NLP and FPT

First year income	NLP	FPT
Premium income	100,000	100,000
Investment income	5,000	5,000
Total income	105,000	105,000
Expense	80,000	80,000
Claim payment	10,000	10,000
Increase in reserves	70,000	0*
Net income	(45,000)	15,000

** Under FPT method, the expense allowance is defined in such way that the end of first year reserve is equal to 0. The proof is illustrated below.*

The definition for expense allowance under FPT method is:

$$ {}_mEA_{x:\overline{n}}^{FTP} = \frac{A_{x+1:\overline{n-1}}}{\ddot{a}_{x+1:\overline{m-1}}} - v \cdot q_x \cdot DB_x $$

DB_x: the death benefit at time 0

Proof for "the end of first policy year reserve is equal to 0 under FPT method":

The end of first-year reserve:

$$ {}_1^mV_{x+1:\overline{n-1}} = {}_1^mVB_{x+1:\overline{n-1}} - {}_1^mVE_{x+1:\overline{n-1}} \quad (1) $$

$$ {}_1^mVB_{x+1:\overline{n-1}} = AB_{x+1:\overline{n-1}} - {}_mPB_{x:\overline{n}} \cdot \ddot{a}_{x+1:\overline{m-1}} = AB_{x+1:\overline{n-1}} - \frac{AB_{x:\overline{n}}}{\ddot{a}_{x:\overline{m}}} \cdot \ddot{a}_{x+1:\overline{m-1}} \quad (2) $$

$$ {}_1^mVE_{x+1:\overline{n-1}} = {}_mPE_x \cdot \ddot{a}_{x+1:\overline{m-1}} = \frac{{}_mEA_{x:n}^{FPT}}{\ddot{a}_{x:\overline{m}}} \cdot \ddot{a}_{x+1:\overline{m-1}} = \left(\frac{AB_{x+1:\overline{n-1}}}{\ddot{a}_{x+1:\overline{m-1}}} - v \cdot q_x \cdot DB_x\right) \cdot \frac{\ddot{a}_{x+1:\overline{m-1}}}{\ddot{a}_{x:\overline{m}}} \quad (3) $$

Substitute (2) and (3) into (1)

$$ {}_1^mV_{x+1:\overline{n-1}} = AB_{x+1:\overline{n-1}} - \frac{AB_{x:\overline{n}}}{\ddot{a}_{x:\overline{m}}} \cdot \ddot{a}_{x+1:\overline{m-1}} - \left(\frac{AB_{x+1:\overline{n-1}}}{\ddot{a}_{x+1:\overline{m-1}}} - v \cdot q_x \cdot DB_x\right) \cdot \frac{\ddot{a}_{x+1:\overline{m-1}}}{\ddot{a}_{x:\overline{m}}} $$

$$ = AB_{x+1:\overline{n-1}} \cdot \left(1 - \frac{1}{\ddot{a}_{x:\overline{m}}}\right) - \frac{\ddot{a}_{x+1:\overline{m-1}}}{\ddot{a}_{x:\overline{m}}} \cdot (AB_{x:\overline{n}} - v \cdot q_x \cdot DB_x) $$

$$= AB_{x+1:\overline{n-1}} \cdot \left(1 - \frac{1}{\ddot{a}_{x:\overline{m}}}\right) - \frac{\ddot{a}_{x+1:\overline{m-1}}}{\ddot{a}_{x:\overline{m}}} \cdot \left(v \cdot q_x \cdot DB_x + AB_{x+1:\overline{n-1}} \cdot v \cdot p_x - v \cdot q_x \cdot DB_x \right)$$

$$= AB_{x+1:\overline{n-1}} \cdot \left(1 - \frac{1}{\ddot{a}_{x:\overline{m}}}\right) - \frac{\ddot{a}_{x+1:\overline{m-1}}}{\ddot{a}_{x:\overline{m}}} \cdot AB_{x+1:\overline{n-1}} \cdot v \cdot p_x$$

$$= AB_{x+1:\overline{n-1}} \cdot \left(1 - \frac{1}{\ddot{a}_{x:\overline{m}}} - \frac{v \cdot p_x \cdot \ddot{a}_{x+1:\overline{m-1}}}{\ddot{a}_{x:\overline{m}}}\right)$$

$$= AB_{x+1:\overline{n-1}} \cdot \left(\frac{\ddot{a}_{x:\overline{m}} - \left(1 + v \cdot p_x \cdot \ddot{a}_{x+1:\overline{m-1}}\right)}{\ddot{a}_{x:\overline{m}}}\right) = 0$$

The same calculation example used in NLP above will be used here under FPT methods:

Assume a 40-year-old male, who bought a 10-pay 20-year endowment policy on 1/1/2000. The 1980 CSO mortality table and 4% interest rate are used for reserve valuation.

For the last year of endowment policies: $A_{x+n-1:\overline{1}} = Death\ Benefit * v$

For the last year of premium payment period:

$$\ddot{a}_{x+m-1:\overline{1}} = 1, \qquad for\ endowment\ policies$$

Retrospective method:

$$A_{x+t:\overline{n-t}} = A_{x+t+1:\overline{n-t-1}} \cdot v \cdot p_{x+t} + DB_{x+t} \cdot v \cdot q_{x+t}, \ \ t < n$$

$$\ddot{a}_{x+t:\overline{m-t}} = \ddot{a}_{x+t+1:\overline{m-t-1}} \cdot v \cdot p_{x+t} + 1, \ \ t < m$$

Benefit reserve:

$${}_t^m VB_x = A_{x+t:\overline{n-t}} - PB * \ddot{a}_{x+t:\overline{m-t}}$$

PB stands for benefit premium and formula is:

$${}_m PB_{x:\overline{n}} = \frac{A_{x:\overline{n}}}{\ddot{a}_{x:\overline{m}}}$$

${}_m PB_{x:\overline{n}}$: The benefit premium for m-pay n-year endowment policy.

Expense reserve:

$$ {}^{m}_{t}VE_x = PE * \ddot{a}_{x+t:\overline{m-t}} $$

PE stands for expense premium and is calculated using the expense allowance:

$$ {}_{m}PE_{x:\overline{n}} = \frac{{}_{m}EA^{FTP}_{x:\overline{n}}}{\ddot{a}_{x:\overline{m}}} $$

${}_{m}PE_{x:\overline{n}}$: The expense premium for m-pay n-year endowment policy;

Net reserve:

$$ {}^{m}_{t}V_x = {}^{m}_{t}VB_x - {}^{m}_{t}VE_x $$

Here shows calculations for expense allowance, PE and PB for the same example:

Assume a 40-year-old male, who bought a 10-pay 20-year endowment policy with $100,000 death benefit on 1/1/2000. The 1980 CSO mortality table and 4% interest rate are used for reserve valuation.

$$ A_{x+1:\overline{n-1}} = 49053; \quad \ddot{a}_{x+1:\overline{m-1}} = 7.65 $$

$$ EA = \frac{A_{x+1:\overline{n-1}}}{\ddot{a}_{x+1:\overline{m-1}}} - v \cdot q_x \cdot DB_x $$

$$ = \frac{49053}{7.65} - 0.0023 * 0.96 * 100{,}000 = 6194 $$

$$ PE = \frac{6193.63}{7.65} = 743 $$

$$ PB = \frac{A_{x+1:\overline{n-1}}}{\ddot{a}_{x+1:\overline{m-1}}} = \frac{47279}{8.34} = 5671 $$

Table 5 Calculation Results of Full-Preliminary Term Reserve Method for a 10-pay 20-year Endowment Policy

Pol Yr	Att. Age	t	q(x)	p(x)	Current Plan			VB	PE	VE	V=VB-VE
					DB(x+t)	A(x)	$\ddot{a}$(x)	NLP Rsv			
1	40	0	0.0023	0.9977	100,000	47,279	8.34	-	743	6,194	-
2	41	1	0.0025	0.9975	100,000	49,053	7.65	5,682	743	5,682	-
3	42	2	0.0027	0.9974	100,000	50,894	6.93	11,588	743	5,149	6,439
4	43	3	0.0029	0.9971	100,000	52,805	6.18	17,732	743	4,595	13,137
5	44	4	0.0031	0.9969	100,000	54,788	5.41	24,122	743	4,017	20,104
6	45	5	0.0033	0.9967	100,000	56,847	4.60	30,772	743	3,416	27,356
7	46	6	0.0036	0.9964	100,000	58,985	3.75	37,694	743	2,789	34,904
8	47	7	0.0039	0.9961	100,000	61,205	2.87	44,901	743	2,136	42,765
9	48	8	0.0042	0.9958	100,000	63,511	1.96	52,410	743	1,454	50,956
10	49	9	0.0045	0.9955	100,000	65,909	1.00	60,238	743	743	59,495
11	50	10	0.0049	0.9951	100,000	68,402	-	68,402	-	-	68,402
12	51	11	0.0054	0.9947	100,000	70,995	-	70,995	-	-	70,995
13	52	12	0.0059	0.9941	100,000	73,695	-	73,695	-	-	73,695
14	53	13	0.0064	0.9936	100,000	76,505	-	76,505	-	-	76,505
15	54	14	0.0071	0.9929	100,000	79,433	-	79,433	-	-	79,433
16	55	15	0.0078	0.9922	100,000	82,486	-	82,486	-	-	82,486
17	56	16	0.0086	0.9914	100,000	85,673	-	85,673	-	-	85,673
18	57	17	0.0095	0.9905	100,000	89,005	-	89,005	-	-	89,005
19	58	18	0.0104	0.9896	100,000	92,494	-	92,494	-	-	92,494
20	59	19	0.0115	0.9885	100,000	96,154	-	96,154	-	-	96,154
21	60	20	0.0126	0.9874	-	-	-	-	-	-	-

7.4 Commissioners Reserve Valuation Method (CRVM)

Commissioners Reserve Valuation Method (CRVM) was defined as the minimum valuation standard to calculate reserve liabilities for life insurance policies by "Accounting Practices and Procedures Manual". It allows insurance companies to use expense allowance to reduce their first year reserves calculated using Net Level Premium Method (NLP). The reduced amount is amortized during the premium payment period. According to the definition, the formula for expense allowance in CRVM valuation is:

$$
\begin{aligned}
{}_mEA_{x:\overline{n}}^{CRVM} &= MIN({}_mEA_{x:\overline{n}}^{FPT}, {}_{20}EA_x^{FPT}) \\
&= MIN(\frac{A_{x+1:\overline{n-1}}}{\ddot{a}_{x+1:\overline{m-1}}} - v \cdot q_x \cdot DB_x, \frac{A_{x+1}}{\ddot{a}_{x+1:\overline{19}}} - v \cdot q_x \cdot DB_x)
\end{aligned}
$$

Where $DB_x = \frac{1}{9}\sum_{s=1}^{s=9} DB_{x+s}^{C}$, DB^C is the death benefit of current plan.

${}_mEA_{x:\overline{n}}^{FPT}$: The expense allowance under full preliminary term method of current policy;

${}_{20}EA_x^{FPT}$: The expense allowance under full preliminary term method of 20-pay whole life policy.

CRVM reserve is determined by the following formula:

$$
{}_t^mV_{x:\overline{n}}^{CRVM} = A_{x+t:\overline{n-t}} - \ddot{a}_{x+t:\overline{m-t}} \cdot \beta \qquad for\ 0 < t < m
$$

Where β is the modified renewal net premium

$$
\beta = \frac{A_{x:\overline{n}} + {}_mEA_{x:\overline{n}}^{CRVM}}{\ddot{a}_{x:\overline{m}}}
$$

In order to calculate expense allowance, the expense allowance under the 20-pay whole life insurance need to determine first. The following table is used to calculate A_{x+1} and $\ddot{a}_{x+1:\overline{19}}$

$$A_{x+1} = 27{,}957; \quad \ddot{a}_{x+1:\overline{19}} = 13.25$$

$$_{20}EA_x^{FPT} = 1890.38 < {}_mEA_{x:\overline{n}}^{FPT} = 6193.63$$

Table 6 Calculation for a 20-pay Whole Life Policy

Pol Yr	Att. Age	t	q(x)	p(x)	20-Pay Whole Life		
					DB(x+t)	A(x)	$\ddot{a}(x)$
1	40	0	0.0023	0.9977	100,000	27,040	13.71
2	41	1	0.0025	0.9975	100,000	27,957	13.25
3	42	2	0.0027	0.9974	100,000	28,899	12.77
4	43	3	0.0029	0.9971	100,000	29,870	12.27
5	44	4	0.0031	0.9969	100,000	30,867	11.76
6	45	5	0.0033	0.9967	100,000	31,892	11.22
7	46	6	0.0036	0.9964	100,000	32,945	10.66
8	47	7	0.0039	0.9961	100,000	34,026	10.09
9	48	8	0.0042	0.9958	100,000	35,136	9.49
10	49	9	0.0045	0.9955	100,000	36,274	8.86
11	50	10	0.0049	0.9951	100,000	37,441	8.22
12	51	11	0.0054	0.9947	100,000	38,637	7.54
13	52	12	0.0059	0.9941	100,000	39,861	6.84
14	53	13	0.0064	0.9936	100,000	41,110	6.11
15	54	14	0.0071	0.9929	100,000	42,384	5.35
16	55	15	0.0078	0.9922	100,000	43,681	4.55
17	56	16	0.0086	0.9914	100,000	44,998	3.72
18	57	17	0.0095	0.9905	100,000	46,334	2.86
19	58	18	0.0104	0.9896	100,000	47,691	1.95
20	59	19	0.0115	0.9885	100,000	49,068	1.00
21	60	20	0.0126	0.9874	100,000	50,463	-
22	61	21	0.0139	0.9861	100,000	51,873	-
23	62	22	0.0154	0.9846	100,000	53,297	-
24	63	23	0.0171	0.9829	100,000	54,731	-
25	64	24	0.0190	0.9810	100,000	56,170	-
26	65	25	0.0211	0.9789	100,000	57,611	-
27	66	26	0.0234	0.9766	100,000	59,050	-
28	67	27	0.0259	0.9741	100,000	60,487	-
29	68	28	0.0285	0.9715	100,000	61,922	-
30	69	29	0.0314	0.9686	100,000	63,354	-

Table 6 Calculation for a 20-pay Whole Life Policy (Continue)

Pol Yr	Att. Age	t	q(x)	p(x)	20-Pay Whole Life		
					DB(x+t)	A(x)	$\ddot{a}$(x)
31	70	30	0.0346	0.9654	100,000	64,783	-
32	71	31	0.0389	0.9611	100,000	66,204	-
33	72	32	0.0426	0.9574	100,000	67,592	-
34	73	33	0.0474	0.9526	100,000	68,975	-
35	74	34	0.0529	0.9471	100,000	70,326	-
36	75	35	0.0588	0.9412	100,000	71,638	-
37	76	36	0.0651	0.9349	100,000	72,911	-
38	77	37	0.0716	0.9284	100,000	74,145	-
39	78	38	0.0785	0.9215	100,000	75,345	-
40	79	39	0.0857	0.9143	100,000	76,516	-
41	80	40	0.0937	0.9063	100,000	77,662	-
42	81	41	0.1025	0.8975	100,000	78,780	-
43	82	42	0.1125	0.8875	100,000	79,868	-
44	83	43	0.1238	0.8762	100,000	80,915	-
45	84	44	0.1361	0.8639	100,000	81,912	-
46	85	45	0.1492	0.8508	100,000	82,855	-
47	86	46	0.1628	0.8372	100,000	83,744	-
48	87	47	0.1768	0.8232	100,000	84,584	-
49	88	48	0.1909	0.8091	100,000	85,383	-
50	89	49	0.2053	0.7947	100,000	86,156	-
51	90	50	0.2202	0.7798	100,000	86,916	-
52	91	51	0.2358	0.7642	100,000	87,680	-
53	92	52	0.2528	0.7473	100,000	88,468	-
54	93	53	0.2716	0.7284	100,000	89,303	-
55	94	54	0.2957	0.7044	100,000	90,218	-
56	95	55	0.3300	0.6700	100,000	91,236	-
57	96	56	0.3846	0.6155	100,000	92,366	-
58	97	57	0.4802	0.5198	100,000	93,599	-
59	98	58	0.6580	0.3420	100,000	94,889	-
60	99	59	1.0000	-	100,000	96,154	-

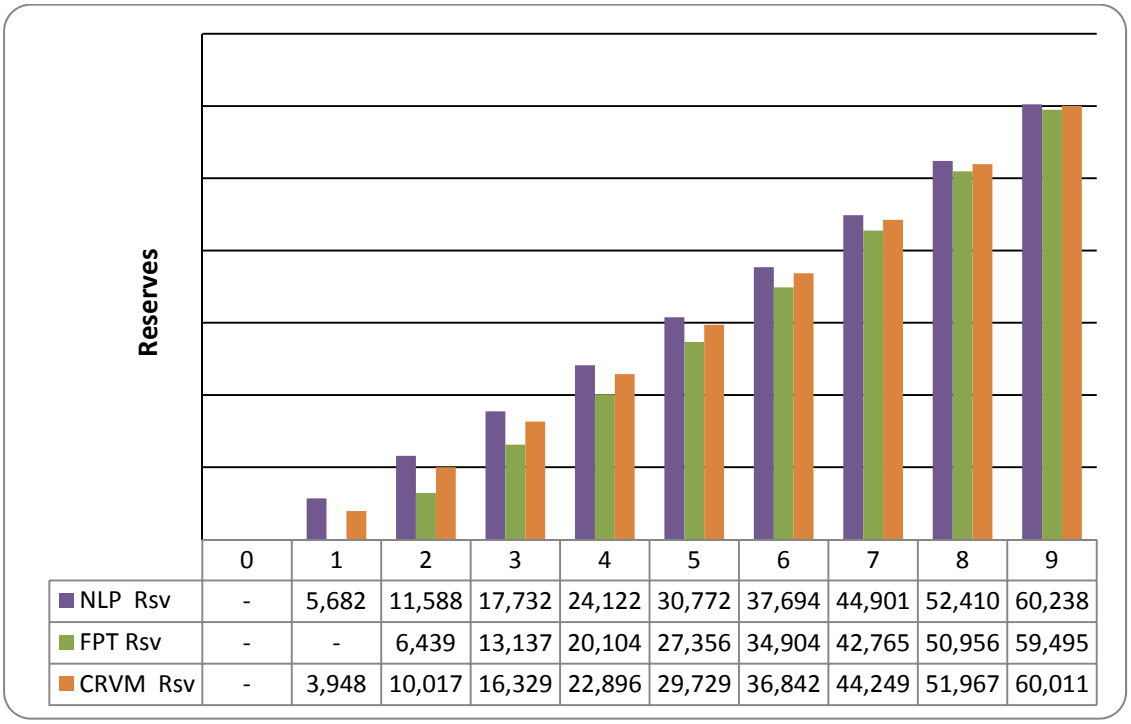
Table 7 Commissioners Reserve Valuation Method (CRVM) for a 10-pay 20-year Endowment Policy

Pol Yr	Att. Age	t	q(x)	p(x)	Current Plan			VB	PE	VE	V=VB-VE
					DB(x+t)	A(x)	$\ddot{a}$(x)	NLP Rsv			
1	40	0	0.0023	0.9977	100,000	47,279	8.34	-	227	1,890	-
2	41	1	0.0025	0.9975	100,000	49,053	7.65	5,682	227	1,734	3,948
3	42	2	0.0027	0.9974	100,000	50,894	6.93	11,588	227	1,572	10,017
4	43	3	0.0029	0.9971	100,000	52,805	6.18	17,732	227	1,402	16,329
5	44	4	0.0031	0.9969	100,000	54,788	5.41	24,122	227	1,226	22,896
6	45	5	0.0033	0.9967	100,000	56,847	4.60	30,772	227	1,043	29,729
7	46	6	0.0036	0.9964	100,000	58,985	3.75	37,694	227	851	36,842
8	47	7	0.0039	0.9961	100,000	61,205	2.87	44,901	227	652	44,249
9	48	8	0.0042	0.9958	100,000	63,511	1.96	52,410	227	444	51,967
10	49	9	0.0045	0.9955	100,000	65,909	1.00	60,238	227	227	60,011
11	50	10	0.0049	0.9951	100,000	68,402	-	68,402	-	-	68,402
12	51	11	0.0054	0.9947	100,000	70,995	-	70,995	-	-	70,995
13	52	12	0.0059	0.9941	100,000	73,695	-	73,695	-	-	73,695
14	53	13	0.0064	0.9936	100,000	76,505	-	76,505	-	-	76,505
15	54	14	0.0071	0.9929	100,000	79,433	-	79,433	-	-	79,433
16	55	15	0.0078	0.9922	100,000	82,486	-	82,486	-	-	82,486
17	56	16	0.0086	0.9914	100,000	85,673	-	85,673	-	-	85,673
18	57	17	0.0095	0.9905	100,000	89,005	-	89,005	-	-	89,005
19	58	18	0.0104	0.9896	100,000	92,494	-	92,494	-	-	92,494
20	59	19	0.0115	0.9885	100,000	96,154	-	96,154	-	-	96,154

NLP gives the largest reserves among these three reserve methods (NLP, FPT, and CRVM). CRVM allows higher 1st terminal reserve than FPT and the slope of reserves is less steep than that of FPT (See Fig. 1). After premium payment period, the reserves are same under three methods.

When expense allowance for 20-pay whole life plane is greater than that for current plan, FPT generates same reserves as CRVM. Otherwise, CRVM has smaller expense allowance and expense premiums than FPT and therefore greater reserves than FPT. Although there are three valuation methods, CRVM is the minimum standard to calculate reserve liabilities and is commonly used in life insurance companies.

Figure 1 Comparison for Reserves by 3 Valuation Methods during Premium Payment Period



	0	1	2	3	4	5	6	7	8	9
NLP Rsv	-	5,682	11,588	17,732	24,122	30,772	37,694	44,901	52,410	60,238
FPT Rsv	-	-	6,439	13,137	20,104	27,356	34,904	42,765	50,956	59,495
CRVM Rsv	-	3,948	10,017	16,329	22,896	29,729	36,842	44,249	51,967	60,011

Table 8 Comparison between NLP, FPT and CRVM

Pol Yr	Att. Age	t	NLP		FPT		CRVM	
			Net Prem	Rsv	Net Prem	Rsv	Net Prem	Rsv
1	40	0	5,671	-	220	-	4,007	-
2	41	1	5,671	5,682	6,414	-	5,898	3,948
3	42	2	5,671	11,588	6,414	6,439	5,898	10,017
4	43	3	5,671	17,732	6,414	13,137	5,898	16,329
5	44	4	5,671	24,122	6,414	20,104	5,898	22,896
6	45	5	5,671	30,772	6,414	27,356	5,898	29,729
7	46	6	5,671	37,694	6,414	34,904	5,898	36,842
8	47	7	5,671	44,901	6,414	42,765	5,898	44,249
9	48	8	5,671	52,410	6,414	50,956	5,898	51,967
10	49	9	5,671	60,238	6,414	59,495	5,898	60,011
11	50	10	-	68,402	-	68,402	-	68,402
12	51	11	-	70,995	-	70,995	-	70,995
13	52	12	-	73,695	-	73,695	-	73,695
14	53	13	-	76,505	-	76,505	-	76,505
15	54	14	-	79,433	-	79,433	-	79,433
16	55	15	-	82,486	-	82,486	-	82,486
17	56	16	-	85,673	-	85,673	-	85,673
18	57	17	-	89,005	-	89,005	-	89,005
19	58	18	-	92,494	-	92,494	-	92,494
20	59	19	-	96,154	-	96,154	-	96,154
21	60	20	-	-	-	-	-	

Chapter Eight: Related Approximations

8.1 Mean Reserve Method

Terminal reserve is the reserve calculated at the end of policy year and initial reserve is the one calculated at the beginning of the policy year. For most cases, actuaries perform the reserve valuations during the policy year and use some approximation to calculate reserve as of valuation date. One method is to use the mean of terminal reserves and initial reserves. Mean reserves assume that the premiums are paid annually at the beginning of the policy year. But in most cases, insurance companies receive premiums monthly, quarterly or semi-annually. Therefore, mean reserve method would include premiums that have not yet been received as of valuation date, which overstates the insurer's liability by assuming that the policy holders would continue to pay their premium and would not cancel or lapse their policies before the end of policy year. Additional asset has to be set aside to offset the "overstated liability" which is also called "deferred premium". Deferred premiums are the amount that will be due after valuation date and before the next policy anniversary date. Deferred premiums are determined by the amount not due on the net premium basis.

The formula for terminal reserve is:

$${}^{m}_{t}V_{x:\overline{n}} = A_{x+t:\overline{n-t}} - {}_{m}PB_{x:\overline{n}} \cdot \ddot{a}_{x+t:\overline{m-t}}$$

The formula for initial reserve at the beginning of t^{th} year is:

$$ {}^{m}_{t-1}V_{x:\overline{n}} + {}_{m}PB_{x:\overline{n}} $$

The mean reserves:

$$ {}^{m}_{t}MV_{x:\overline{n}} = \frac{{}^{m}_{t}V_{x:\overline{n}} + {}^{m}_{t-1}V_{x:\overline{n}} + {}_{m}PB_{x:\overline{n}}}{2} $$

8.2 Mid-Terminal Method

Mid-terminal reserves equal to the average of the terminal reserves on the previous and current valuation year.

$$ \frac{{}^{m}_{t}V_{x:\overline{n}} + {}^{m}_{t-1}V_{x:\overline{n}}}{2} $$

Mid-terminal reserve method is commonly used for weekly premium business because it understates the policy reserve.

Chapter Nine: Deficiency Reserves

The gross premium is the amount collected by the insurer based on life insurance contract and counted as the premium income in the balance sheet. Gross premium is calculated based on the best estimate of mortality rates, investment income and policy expense. The net premium is the amount calculated by using interest rate and mortality table which are defined in statutory laws. When the gross premium is less than net premium, an additional reserve need to set up , which is called “Deficiency reserve”. Deficiency reserve is equal to present value of gross premium less net premium in the future.

$$ {}_tV_{x:\overline{n}}^{DEF} = \left(A_{x+t:\overline{n-t}} - {}_mGP_{x:\overline{n}} \cdot \ddot{a}_{x+t:\overline{n-t}}\right) - \left(A_{x+t:\overline{n-t}} - {}_mNP_{x:\overline{n}} \cdot \ddot{a}_{x+t:\overline{n-t}}\right) $$

Mortality table and interest are defined by the minimum standards allowed by SVL.

Chapter Ten: Conclusion

Statutory reserves are the amount of asset life insurance companies must to hold in order to guarantee the payment of future benefit. It evaluates the minimum liabilities of life policies under the regulation laws in the United States. Statutory reserves emphasize on the financial solvency of life insurance companies and therefore provide more conservative reserves than GAAP.

Bibliography

Louis J. Lomvardi, 2006. *Valuation of Life Insurance Liabilities: Establishing Reserves for Life Insurance Policies and Annuity Contracts*, 4th Edition.

Albert E.Easton, Timothy F. Harris, 2007. *Actuarial Aspects of Individual Life Insurance and Annuity Contracts*, 2nd Edition.

Mark A. Tullis, Philip K. Polkinghorn, *Valuation of Life Insurance Liabilities*, 3rd Edition.

Society of Actuaries, 1981. *Transactions*, Volume XXXIII.

National Association of Insurance Commissioners, 2003. *Accounting Practices And Procedures Manual*, Volume I, Statements of Statutory Accounting Principles, Appendices A through D.

R.Thomas Herget, Frank Buck, et al., 2000, *U.S. GAAP for life insurers*, 1st Edition.

Vita

Xiaojie Wang received her Degree of Bachelors of Science in 2007 in Chemistry from Nanjing University, Nanjing, China. She joined the Graduate School of Chemistry at University of Texas at Austin in fall 2007 and transferred to the Actuarial Graduate Program in fall 2009. She interned at Life of the Southwest Company during summer of 2010 and interned at New Era Life Insurance Companies from December 2010 to May 2011. At the time when this report was written, she had received the full-time offer from New Era Life Insurance Companies.

Permanent Address: 22400 Westheimer Parkway, Apt.803, Katy, 77450

This report was typed by Xiaojie Wang.